Radiation Resistance

- ***Radiation Resistance*** is the portion of the antenna's impedance that results in power radiated into space (i.e., the effective resistance that is related to the power radiated by the antenna.
- Varies with antenna length. Resistance increases as the λ increases

Effective Radiated Power (ERP)

- *ERP* is the power input value and the gain of the antenna multiplied together
  - *dBi* = isotropic radiator gain
  - *dBd* = dipole antenna gain

Beamwidth

- *Beamwidth* is the angular separation of the half-power points of the radiated pattern

Antenna Gain

- **Antenna gain**
  - Power output, in a particular direction, compared to that produced in any direction by a perfect omnidirectional antenna (isotropic antenna)
- Effective area
  - Related to physical size and shape of antenna

Antenna Gain

- *Antenna gain* is the measure in dB how much more power an antenna will radiate in a certain direction with respect to that which would be radiated by a reference antenna

Antenna Gain

- Relationship between antenna gain and effective area

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi f^2 A_e}{c^2}$$

  - G = antenna gain
  - A_e = effective area
  - f = carrier frequency
  - c = speed of light (» 3 ´ 10^8 m/s)
  - λ = carrier wavelength

Antennas

- Radiated Power
- Radiation Pattern
  - Beamwidth
  - Pattern Solid Angle
  - Directivity
  - Efficiency
  - Gain

Antennas – Radiation Power

Let us consider a transmitting antenna (transmitter) is located at the origin of a spherical coordinate system.
In the far-field, the radiated waves resemble plane waves propagating in the radiation direction and time-harmonic fields can be related by the chapter 5 equations.

Electric and Magnetic Fields:

$$\mathbf{E}_s = -\eta_o \mathbf{a}_r \times \mathbf{H}_s$$

$$and$$

$$\mathbf{H}_s = \frac{1}{\eta_o} \mathbf{a}_r \times \mathbf{E}_s$$

The time-averaged power density vector of the wave is found by the Poynting Theorem

Power Density:

$$\mathbf{P}(r,\theta,\phi) = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \mathbf{H}_s^*\right]$$

$$\mathbf{P}(r,\theta,\phi) = P(r,\theta,\phi)\,\mathbf{a}_\mathbf{r}$$

The total power radiated by the antenna is found by integrating over a closed spherical surface,

Radiated Power:

$$P_{rad} = \oiint \mathbf{P}(r,\theta,\phi) \cdot d\mathbf{S} = \int\int P(r,\theta,\phi)\, r^2 \sin\theta \, d\theta \, d\phi$$

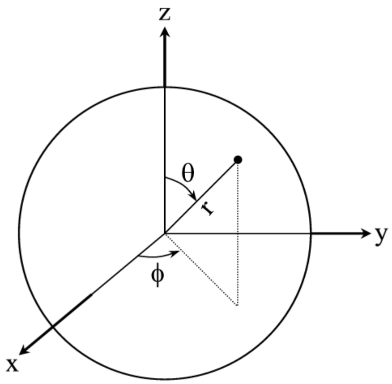
Antennas – Radiation Patterns

Radiation patterns usually indicate either electric field intensity or power intensity. Magnetic field intensity has the same radiation pattern as the electric field intensity, related by η_o

It is customary to divide the field or power component by its maximum value and to plot a normalized function

Normalized radiation intensity:

$$P_n(\theta,\phi) = \frac{P(r,\theta,\phi)}{P_{\max}}$$



Isotropic antenna: The antenna radiates electromagnetic waves equally in all directions.

$$P_n(\theta,\phi)_{iso} = 1$$

Radian And Steradian

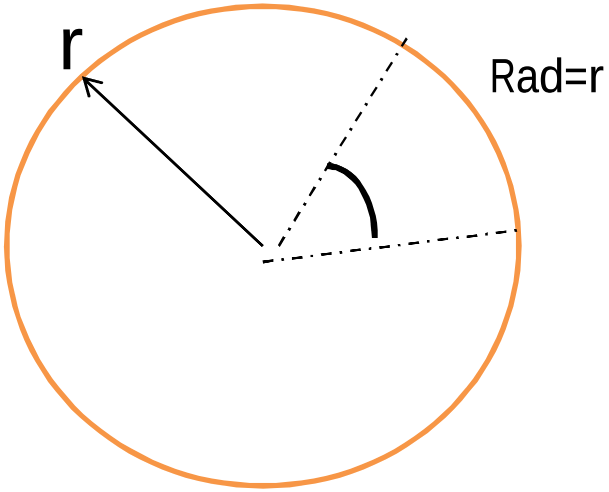
- **Radian**:

The plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r. OR

It is the angle subtended by an arc along the perimeter of the circle with length equal to the radius.

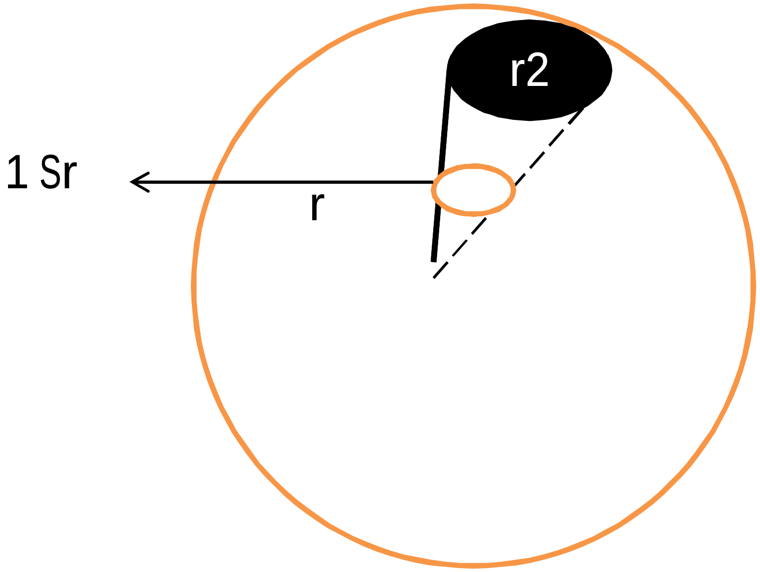
Circumference= **2π X r**

Full circle consists of 2π rad



Steradian(Sr)

- **The measure of solid angle is Sr.**
- ***The solid angle with its vertex at the center of the sphere of radius r that is subtended by a spherical surface area of r*** ^{2}
- **OR**
- One steradian (sr) is subtended by an area r2 at the surface of a sphere of radius r.



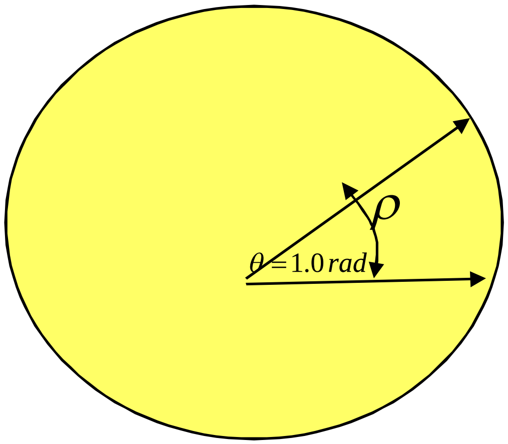
- Area of sphere=$4\pi r^2 = 4\pi \times r^2$

If each r2 area occupies =1 steradian

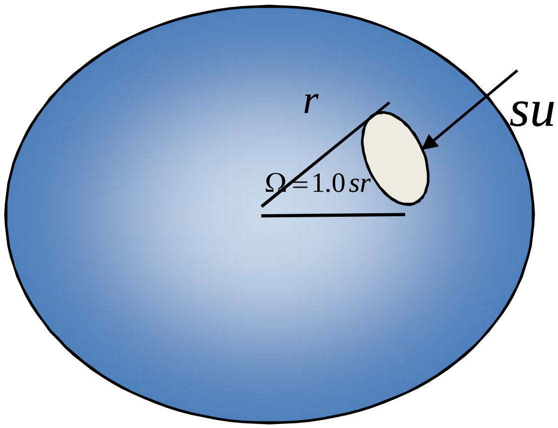
Full sphere consists of 4 π steradians.

Incremental area (ds) & solid angle (dΩ)

Aside on Solid Angles



$$arclength = \rho$$



$$surface\ area = r^2$$

$$total\ \ circumfran\ \ ce\ = 2\pi\ \ radians$$

$$total\ surface\ area = S_o = 4\pi r^2 = \Omega r^2$$

$$\Omega = \frac{S_o}{r^2}\ \ sr$$

infinitesimal area
of surface of sphere

$$ds = r^2 \sin(\theta)\, d\theta\, d\phi$$

$$d\Omega = \frac{ds}{r^2} = \sin(\theta)\, d\theta\, d\phi$$

Incremental area (ds) & solid angle (dΩ)

Antenna Pattern Solid Angle:

A *differential solid angle*, dΩ, in sr, is defined as

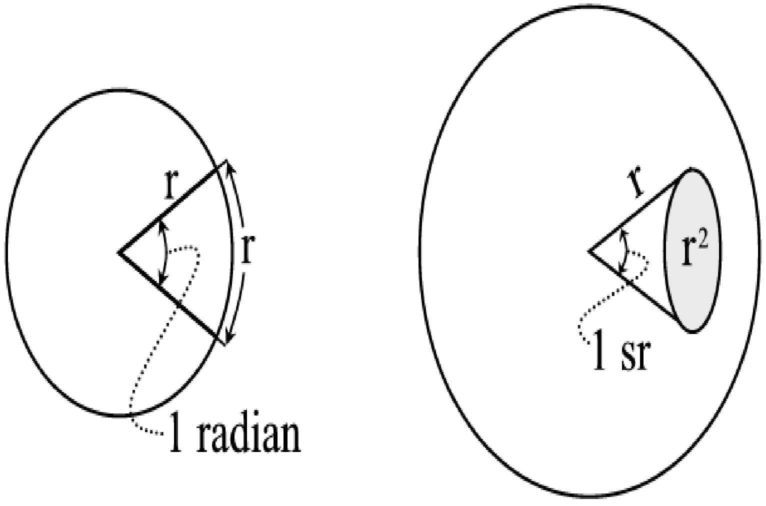
$$d\Omega = \sin\theta d\theta d\phi.$$

For a sphere, the solid angle is found by integrating

$$\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = 4\pi(sr).$$

An antenna's pattern solid angle,

$$\boxed{\Omega_p = \iint P_n(\theta,\phi)\,d\Omega}$$



(a) (b)

All of the radiation emitted by the antenna is concentrated in a cone of solid angle Ω_p over which the radiation is constant and equal to the antenna's maximum radiation value.

Beam Area

- ***The solid angle through which all of the power radiated by the antenna would flow if $P\ (\vartheta,\Phi)$ maintained its maximum value over ΩA and was zero elsewhere.***

$$\Omega_A = \iint_{4\pi} Pn(\vartheta,\Phi)\, d\,\Omega\ (Sr)$$

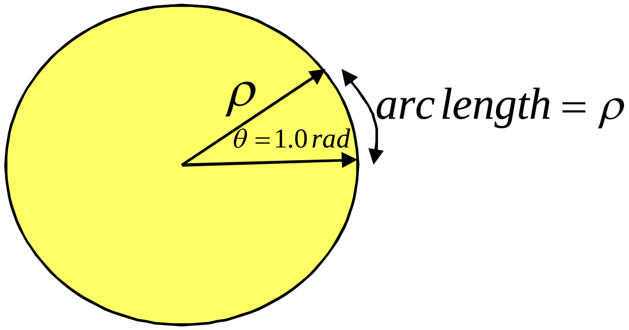
- Where $d\ \Omega = \sin\theta d\theta\ d\Phi\ (Sr)$

And $ds = r^2 \sin\theta d\theta\ d\Phi$

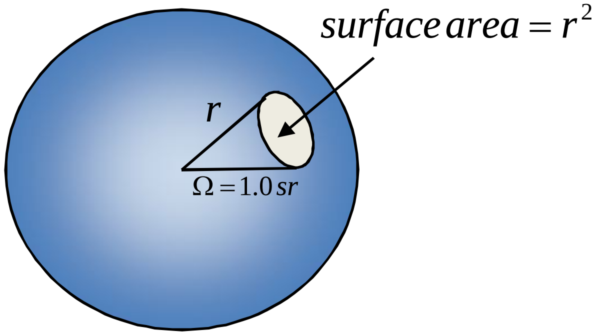
Radiation Intensity

Radiation intensity in a given direction is the power per unit solid angle radiated in this direction by the antenna.

Aside on Solid Angles



$total\,circumfrance = 2\pi\;\;radians$



$$total\;surface\;area = S_o = 4\pi\,r^2 = \Omega\,r^2$$

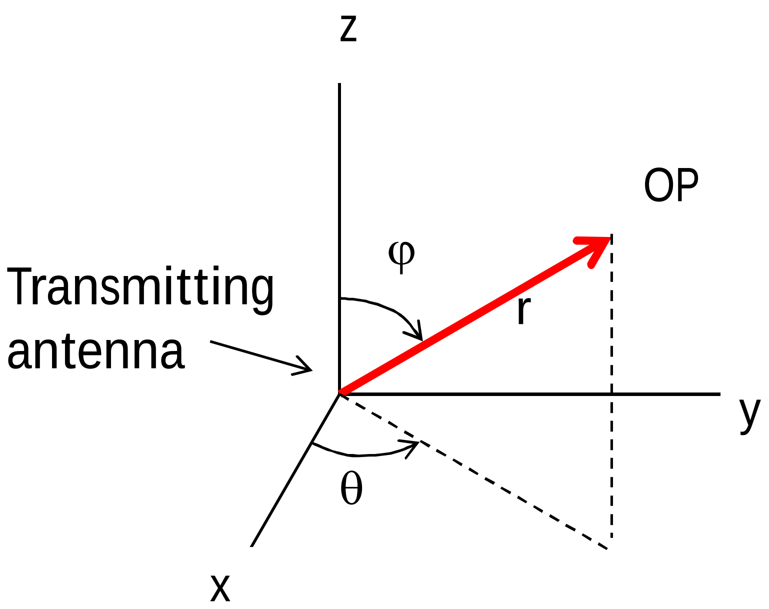
$$\Omega = \frac{S_o}{r^2}\;\;sr$$

infinitesimal area of surface of sphere

$$ds = r^2\sin(\theta)\,d\theta\,d\phi$$

$$d\Omega = \frac{ds}{r^2} = \sin(\theta)\,d\theta\,d\phi$$

Radiation Intensity



Distance (r) is very large

- ***measure of the ability of an antenna to concentrate radiated power in a particular direction***

- **Radiation intensity = Power per steradian =**
  = $\Phi(\theta,\varphi)$ [watts/steradian]

Radiation Intensity

Radiation intensity in a given direction is the power per unit solid angle radiated in this direction by the antenna.

$$U = \frac{dP_{rad}^{tot}}{d\Omega} \; W/sr \qquad \Rightarrow \qquad P_{rad}^{tot} = \oiint_{4\pi} U \; d\Omega$$

$$P_{rad} = \frac{dP_{rad}^{tot}}{ds} \; W/m^2 \qquad \Rightarrow \qquad P_{rad}^{tot} = \oiint P_{rad} \, ds$$

$$\boxed{U = r^2 P_{rad}}$$

since $P_{rad}(\theta,\phi,r)$ decays as $1/r^2$ in the far field

$U(\theta, \phi)$ will be independent of r

The power pattern is a trace of the function $|U(\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\bar{U}(\theta,\varphi)$.

Radiation Intensity

The power pattern is a trace of the function $|U(\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\overline{U}(\theta,\varphi)$.

$$P_{rad}(\theta,\varphi,r)=\frac{1}{2}\tilde{E}\times\tilde{H}^{*}=\frac{1}{2\eta}\left|\tilde{E}\right|^{2}=\frac{1}{2\eta}\left|E_{\theta}{}^{2}+E_{\varphi}{}^{2}\right|$$

$$U(\theta,\varphi)=\frac{r^{2}}{2\eta}\left|E_{\theta}{}^{2}+E_{\varphi}{}^{2}\right|$$

$$\overline{U}(\theta,\varphi)=\frac{U(\theta,\varphi)}{U_{\max}}$$

Radiation Intensity Ex

1. Isotropic radiator

$$P_{rad}(\theta,\varphi,r)=\frac{P_{rad}^{tot}}{4\pi\, r^2}$$

$$U(\theta,\varphi)=r^2\,P_{rad}(\theta,\varphi,r)=\frac{P_{rad}^{tot}}{4\pi}=const$$

$$\overline{U}(\theta,\varphi)=\frac{U(\theta,\varphi)}{U_{\max}}=1.0$$

2. Hertzian Dipole

$$E_\theta(\theta,\varphi,r)=j\eta\frac{\beta\,\Delta l\,I_0\,e^{-j\beta r}}{4\pi\,r}\sin(\theta)$$

$$E_\phi(\theta,\varphi,r)=0$$

$$U(\theta,\varphi)=r^2\frac{1}{2\eta}\left|E_\theta{}^2+E_\phi{}^2\right|=r^2\frac{1}{2\eta}\cdot\left|\eta\frac{\beta\,\Delta l\,I_0\,e^{-j\beta r}}{4\pi\,r}\sin(\theta)\right|^2=\frac{\eta}{2}\left(\frac{\beta\,\Delta l\,I_0}{4\pi}\right)^2\sin^2(\theta)$$

$$\overline{U}(\theta,\varphi)=\frac{U(\theta,\varphi)}{U_{\max}}=\sin^2(\theta)$$

Radiation resistance

- Antenna presents an impedance at its terminals

$$Z_A = R_A + jX_A$$

- Resistive part is radiation resistance plus loss resistance

$$R_A = R_R + R_L$$

The radiation resistance does not correspond to a real resistor present in the antenna but to the resistance of space coupled via the beam to the antenna terminals.